MODEL QUESTION PAPER

BF1

I Semester B.TECH Examination, August 2011
MATHEMATICS – I

Time: 3 Hours Max. Marks: 75

GROUP A : Answer any three questions.

Q.1 Evaluate: $\lim_{x\to\infty} \frac{5x^3 + 6x^2 + 1}{7x^3 + 8x - 5}$.

Q.2 Differentiate $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Q.3 Find the volume of the solid obtained by the revolution of the loop of the curve $(a-x)y^2 = (a+x)x^2$ about the x-axis.

Q.4 A cylindrical tank is to be made, closed at each end, to hold $(25)^{\frac{1}{7}}$ liters of water. Find its length and radius if the quantity of iron plate required is to be a minimum?

Q.5 Find the area bounded by the curves $y^2 = 9x$ and $x^2 = 9y$.

GROUP B : Answer any three questions.

Q.6 Evaluate: $\int e^{4x}.\cos 2x.\cos 4x\, dx$.

Q.7 Evaluate i) $e^x \frac{1+\sin x}{1+\cos x}$ ii) $e^x\left(\frac{x+2}{x+4}\right)^2$

Q.8 Find $\frac{dy}{dx}$ if $x^m \cdot y^n = (x+y)^{m+n}$

Q.9 If $ax^2 + 2hxy + by^2 = 0$ Prove that $\frac{d^2y}{dx^2} = 0$

Q.10 Show that

$$e^x \cos x = 1 + x - \frac{2x^3}{3!} - \frac{2^2x^4}{4!} - \frac{2^2x^5}{5!} + \frac{2^3x^7}{7!} + \ldots.$$

GROUP C: All Questions are Compulsory.

Q.11 Fill in the blanks

(i) If $x = f(t)$, $y = g(t)$, then $\frac{dy}{dx}$ = _____.

(ii) $\lim_{x\to 0}(\cot x)^{\sin x}$ =_____

(iii) By Rolle's Theorem: If a function $f(x)$ is continuous in the closed interval $a \le x \le b$, has a derivative at all the points of the open interval $a < x < b$, and $f(a) = f(b) = 0$, then there is at least one value $x = c$ between a and b such that _____.

(iv) The n^{th} differential coefficient of $\log x$ is _____.

(v) If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)\,dx =$ _____.

Q.12 Multiple choice question.

(i) The areas bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is _____.

(a) $\sqrt{\pi ab}$ (b) $\frac{1}{\sqrt{\pi ab}}$

(c) πab (d) $\pi\sqrt{ab}$.

(ii) The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is _____.

(a) $\frac{1}{\sqrt{1+x^2}}$ (b) $\frac{1+x^2}{2}$

(c) $\frac{2}{1+x^2}$ (d) $\sqrt{1+x^2}$.

(iii) The differential coefficient of $10^{\log \sin x}$ is _____.

(a) $10^{\log \sin x}$ (b) 0

(c) 1 (d) $10^{\log \sin x}$. tan$x$. log10.

(iv) The value of $\int_0^{\pi/2} \sin^8 x\,dx$ is_____.

(a) $\frac{5\pi}{56}$ (b) $\frac{35}{128}$

(c) $\frac{5\pi}{256}$ (d) $\frac{35\pi}{256}$.

(v) $\int \operatorname{cosec} x\,dx =$ _____

(a) $\log(\operatorname{cosec} x - \cot x)$ (b) $\cot x$

(c) $\log(\operatorname{cosec} x + \cot x)$ (d) $\sec x$

Q.13 True or false

(i) The process of finding an area is called quadrature.

(ii) $\frac{d}{dx}\left(a^x\right) = \frac{a^x}{\log_e a}$.

(iii) $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \frac{3}{2}$

(iv) Given $f(x) = x^2 + 2x - 3$, $g(x) = x^2 - 4x + 6$ & $0 \le x \le 1$, the value $c = \frac{1}{2}$ satisfies the Cauchy's mean value theorem.

(v) $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$

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